

Non-Abelian magnetic monopole in a Bose-Einstein condensate

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Recently, an effective non-Abelian magnetic field with a topology of a monopole was shown to emerge from the adiabatic motion of multilevel atoms in spatially varying laser fields [J. Ruseckas *et al.*, Phys. Rev. Lett. **95**, 010404 (2005)]. We study this monopole in a Bose-Einstein condensate (BEC) of degenerate dressed states and find that the topological charge of the pseudospin cancels the monopole charge resulting in a vanishing gauge invariant charge. As a function of the laser wavelength, different stationary states are classified in terms of their effect to the monopole part of the magnetic field and a cross-over to vortex ground state is observed.

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Introduction. — Existence of an isolated point source of magnetic field, that is, a magnetic monopole, has been an intriguing question ever since Dirac proposed that the existence of a magnetic monopole leads in a natural way to the quantization of electric charge [1]. The magnetic monopole considered by Dirac is intrinsically a singular object with an string-like singularity attached to it. In the seminal work of 't Hooft and Polyakov, non-singular monopole configurations were shown to arise in the context of non-Abelian gauge theories [2, 3]. Unfortunately, these kind of monopoles are expected to be extremely heavy which renders their experimental detection difficult. Despite ingenious theoretical and experimental work in the fields of high energy physics, cosmology, and condensed matter physics, convincing evidence of the existence of stable magnetic monopoles is still lacking [4].

Advances in trapping and manipulating degenerate quantum gases have revealed the potential of the cold atom systems to serve as quantum simulators for ideas beyond the usual condensed matter phenomena. In particular, it has been proposed that non-Abelian gauge potentials are realized in the effective description of atoms with degenerate internal degrees of freedom coupled to spatially varying laser fields [5, 6, 7, 8]. Based on the earlier work by Wilczek and Zee [9], Ruseckas *et al.* [6] have shown that non-Abelian gauge potentials describe the off-diagonal couplings between the degenerate dressed states in atoms with multiple degenerate internal states. As a specific example, a $U(2)$ gauge potential corresponding to a non-Abelian magnetic monopole was constructed [6].

Previous monopole studies in Bose-Einstein condensates (BECs) have concentrated on systems where the monopole state occurs without any gauge potential and stems essentially from the non-linear interactions between atoms in different hyperfine spin states [10]. This renders such monopoles typically energetically unfavorable [10]. On the other hand, atom-laser interaction induced Abelian magnetic monopoles and non-Abelian monopoles in optical lattices have been investigated re-

cently [11]. In this Letter, we consider an isolated non-Abelian magnetic monopole in a Bose-Einstein condensate, which is similar to the monopoles in the Yang-Mills-Higgs model [12]. We study the lowest energy states of the system and show that they can be classified according to the topological charge of the pseudospin texture generated by gauge transformations. On the other hand, we find the gauge invariant total charge of the system to be always zero.

The model. — Consider a four-level tripod scheme, in which degenerate atomic states $\{|1\rangle, |2\rangle, |3\rangle\}$ are excited to a common virtual state $|0\rangle$, see [6]. In the rotating wave approximation the atom-light interaction Hamiltonian has two degenerate dark states $|\chi_1\rangle$ and $|\chi_2\rangle$ corresponding to zero eigenvalue, and two bright states $|\chi_3\rangle$ and $|\chi_4\rangle$ with a non-zero projection to the radiatively decaying excited level $|0\rangle$. Expressing the full quantum state $|\Phi\rangle = \sum_{k=0}^3 \zeta_k(\mathbf{r})|k\rangle$ in the basis of the dressed states as $|\Phi\rangle = \sum_{m=1}^4 \psi_m(\mathbf{r})|\chi_m(\mathbf{r})\rangle$ gives rise to a unitary transformation $\mathcal{U}_{mk}(\mathbf{r}) = \langle\chi_m(\mathbf{r})|k\rangle$ between vectors $\Psi = (\psi_1 \cdots \psi_4)^T$ and $\zeta = (\zeta_0 \cdots \zeta_3)^T$ as $\psi_m(\mathbf{r}) = \sum_{k=0}^3 \mathcal{U}_{mk}(\mathbf{r})\zeta_k(\mathbf{r})$. If transitions from the degenerate dark states to bright states are neglected one obtains an effective single-particle Hamiltonian for the dark states

$$\hat{H}_1 = \int d\mathbf{r} \left[\frac{\hbar^2}{2m} (D_\mu \hat{\psi})^\dagger (D_\mu \hat{\psi}) + \hat{\psi}^\dagger (V_{\text{ext}} + \Phi) \hat{\psi} \right], \quad (1)$$

where $D_\mu = \partial_\mu - iA_\mu$ and $\hat{\psi} = (\hat{\psi}_1, \hat{\psi}_2)^T$, see Ref. [6]. The Berry connection $\mathbf{A} = A_\mu \mathbf{e}_\mu$ and the scalar potential Φ transform as $\mathbf{A} \rightarrow U \mathbf{A} U^\dagger - i(\nabla U)U^\dagger$ and $\Phi \rightarrow U \Phi U^\dagger$ under a local change of basis given by a unitary matrix $U(\mathbf{r})$. Furthermore, we denote $A_\mu = A_\mu^\alpha \sigma^\alpha$ and $\sigma^\alpha = (1, \sigma^x, \sigma^y, \sigma^z)$ [19]. The external potential that confines the atoms is given by V_{ext} .

The non-Abelian magnetic monopole was shown to arise from two circularly polarized beams and a linearly polarized beam yielding [6]

$$\mathbf{A} = -\frac{\cos \vartheta}{r \sin \vartheta} \mathbf{e}_\varphi \sigma^x + \frac{1}{2}(k\mathbf{e}_z - k'\mathbf{e}_x) \times [(1 + \cos^2 \vartheta) \mathbb{1} + \sin^2 \vartheta \sigma^z], \quad (2)$$

$$\Phi = \frac{\hbar^2}{2m} \left[\frac{1}{r^2} \mathbb{1} - \frac{k'}{2r} \sin \varphi \sin 2\vartheta \sigma^x + \frac{k^2 + k'^2}{8} \sin^2 2\vartheta (\mathbb{1} - \sigma^z) \right], \quad (3)$$

where (r, φ, ϑ) denote the spherical coordinates. As pointed out in [6], the effective magnetic field $B_\mu = \frac{1}{2} \varepsilon_{\mu\nu\lambda} F_{\nu\lambda}^\alpha \sigma^\alpha$ is of the form $\mathbf{B} = 1/r^2 \mathbf{e}_r \sigma^x + \dots$ in which the omitted “non-monopole” terms do not generate a net flux through a closed surface enclosing the monopole. The field strength tensor $F_{\mu\nu}^\alpha$ is given by $F_{\mu\nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + 2\varepsilon^{abc} A_\mu^b A_\nu^c$ and $F_{\mu\nu}^0 = \partial_\mu A_\nu^0 - \partial_\nu A_\mu^0$, see [19].

Let us consider the atom-laser and the atom-atom interactions in the original basis $\{|k\rangle\}$, $k = 0, \dots, 3$. For a system consisting of, e.g., ^{87}Rb atoms with hyperfine spin F , the set of degenerate internal states $\{|1\rangle, |2\rangle, |3\rangle\}$ can be taken to be the Zeeman sublevels $|F = 1, m_F\rangle$, $m_F = 1, -1, 0$ at the $5S_{1/2}$ multiplet, and the excited state e.g. $|F = 0, m_F = 0\rangle$ at the $5P_{3/2}$ multiplet. For ^{87}Rb the scattering lengths in the channels corresponding to total hyperfine spin $f = 0$ and $f = 2$ of the two scattering particles are almost equal [13], and hence we neglect the spin-dependent part of interaction between states $|F = 1, m_F\rangle$, see [14]. Furthermore, similarly to Ref. [8] we assume that the population of the two bright states $|\chi_3\rangle$ and $|\chi_4\rangle$ is vanishingly small compared to the population of the dark states $|\chi_1\rangle$ and $|\chi_2\rangle$. This is, in fact, a non-trivial assumption since for a three-level Λ -system it has been shown that the dark state can be unstable under small deviations, leading to non-vanishing population of the bright state [15]. Similar analysis for the tripod system considered here has yet to be performed. Using the unitary transformation $\mathcal{U}_{mk}(\mathbf{r})$ and the Rabi frequencies [6] $\Omega_{1,2}(\mathbf{r}) = \Omega_0 \frac{\rho}{\sqrt{2R}} e^{i(kz \mp \varphi)}$, $\Omega_3(\mathbf{r}) = \Omega_0 \frac{z}{R} e^{ik'x}$ corresponding to the monopole field, one obtains an effective Hamiltonian $\hat{H}_{AA} = \frac{c_0}{2} \int d\mathbf{r} (\hat{\psi}^\dagger \hat{\psi})^2$ describing the interactions in the dark state manifold.

In the absence of light induced gauge potentials Bose-Einstein condensation takes place at low enough temperatures. On the other hand, the existence of artificial gauge potentials can alter the formation of BEC, leading to fragmentation [16]. Although strictly speaking the mean field approach used below cannot treat effects such as fragmentation, even in the absence of a true condensate the mean field states can be used to construct approximations for the true ground state as well as to interpret the possible outcomes of a particular experimental realization [17]. Replacing $\hat{\psi}$ by $\langle \hat{\psi} \rangle = \psi$ we obtain a

mean field Hamiltonian

$$\mathcal{H} = \int d\mathbf{r} \left[\frac{\hbar^2}{2m} (D_\mu \psi)^\dagger (D_\mu \psi) + \psi^\dagger (V_{\text{ext}} + \Phi) \psi + \frac{c_0}{2} |\psi|^4 \right]. \quad (4)$$

The atom-laser interaction is diagonal in the basis consisting of the dressed states $\{|\chi(\mathbf{r})\rangle\}$. The dark states correspond to a zero eigenvalue and hence the atom-laser interaction does not appear in Eq. (4). For optically confined spin-1 bosons such as ^{87}Rb , the external trapping potential V_{ext} can be treated as a scalar function [14] and the model in Eq. (4) is $U(2)$ gauge invariant.

Gauge invariance. — In the Abelian case, the magnetic monopole is accompanied with singular filament, the Dirac string, that extends outwards from the monopole. For non-Abelian monopoles it has been pointed out that the Dirac string can be removed by a suitable gauge transformation [12]. Similar line of reasoning applies also here and by applying a gauge transformation $U_0(\mathbf{r}) = e^{i\varphi\sigma^x} e^{-i\vartheta\sigma^y/2}$ one can remove the divergent part of \mathbf{A} . The transformed vector potential is given by

$$\mathbf{A}' = \frac{1}{r} \mathbf{e}_\varphi b^a \sigma^a - \frac{1}{2r} \mathbf{e}_\vartheta [\cos(2\varphi) \sigma^y - \sin(2\varphi) \sigma^z] + \frac{1}{2} (k\mathbf{e}_z - k'\mathbf{e}_x) [(1 + \cos^2 \vartheta) \mathbb{1} + \sin^2 \vartheta b^a \sigma^a], \quad (5)$$

with $\mathbf{b} = (\sin \vartheta, \sin(2\varphi) \cos \vartheta, \cos(2\varphi) \cos \vartheta)$. The scalar potential transforms according to $\Phi' = U_0 \Phi U_0^\dagger$. It should be noted that \mathbf{A}' is not well-defined at the z axis although the gauge transformation removed the explicit divergence. The scalar potential Φ' on the other hand, remains well-defined at the z axis.

The topological structure of the condensate can be analyzed by studying the pseudospin

$$\mathbf{s}(\mathbf{r}) = \psi^\dagger(\mathbf{r}) \boldsymbol{\sigma} \psi(\mathbf{r}). \quad (6)$$

We note that \mathbf{s} is not itself gauge invariant, but transforms under any gauge transformation U as $s^a \rightarrow u^{ab} s^b$ where $U^\dagger \sigma^a U = u^{ab} \sigma^b$. An explicit computation of the matrix elements u^{ab} shows that the gauge transformation of the pseudospin s^a can be written as

$$s^a \sigma^a \rightarrow U s^a \sigma^a U^\dagger. \quad (7)$$

We define the covariant derivative of the pseudospin by

$$\mathcal{D}_\mu s^a = \psi^\dagger \sigma^a D_\mu \psi + \text{c.c.} = \partial_\mu s^a + 2\varepsilon^{abc} A_\mu^b s^c, \quad (8)$$

from which we see that $\mathcal{D}_\mu s^a$ transforms as $\mathcal{D}_\mu s^a \rightarrow u^{ab} \mathcal{D}_\mu s^b$. Using identities $2|\mathbf{s}|^2 = \text{tr}(s^a \sigma^a s^b \sigma^b)$, $2\varepsilon^{abc} s^a \mathcal{D}_\mu s^b \mathcal{D}_\nu s^c = -i \text{tr}[s^a \sigma^a (\mathcal{D}_\mu s^b \sigma^b (\mathcal{D}_\nu s^c \sigma^c))]$, and Eq. (7) one observes that

$$G_{\mu\nu} = \frac{1}{|\mathbf{s}|} F_{\mu\nu}^a s^a - \frac{1}{2|\mathbf{s}|^3} \varepsilon^{abc} s^a \mathcal{D}_\mu s^b \mathcal{D}_\nu s^c, \quad (9)$$

is invariant under $U(2)$ gauge transformations of ψ and A_μ . Tensor $G_{\mu\nu}$ is the “electromagnetic” tensor introduced by 't Hooft in the studies of monopoles in unified

gauge theories [2], although here \mathbf{s} is not the fundamental variable but defined by Eq. (6).

Using a unit vector field $\hat{\mathbf{n}} = \mathbf{s}/|\mathbf{s}|$ one can write $G_{\mu\nu}$ in the form [12] $G_{\mu\nu} = M_{\mu\nu} - H_{\mu\nu}$ where

$$M_{\mu\nu} = \partial_\mu(\hat{n}^a A_\nu^a) - \partial_\nu(\hat{n}^a A_\mu^a), \quad (10)$$

$$H_{\mu\nu} = \frac{1}{2}\varepsilon^{abc}\hat{n}^a\partial_\mu\hat{n}^b\partial_\nu\hat{n}^c. \quad (11)$$

In particular, Eqs. (10)–(11) imply that $G_{\mu\nu}$ is linear with respect to the gauge potential A_μ^a . This enables us to study separately the terms in the gauge potential which are responsible for the monopole and consider the terms depending on k or k' as a background field for the monopole configuration, see Eq. (5). Using the tensor $G_{\mu\nu}$ we define magnetic charge density $J = \varepsilon_{\mu\nu\lambda}\partial_\mu G_{\nu\lambda}$, and with the Gauss' theorem the total magnetic charge can be written as [12]

$$Q = \frac{1}{8\pi} \int_{S^\infty} d\tau_\mu \varepsilon_{\mu\nu\lambda} [M_{\nu\lambda} - H_{\nu\lambda}] = Q_M - Q_S. \quad (12)$$

where $d\tau_\mu$ is the surface element on S^∞ . The factor of 2 in the derivative of s^a [Eq. (8)] implies that Q_S is quantized in the units of $\frac{1}{2}$, that is, Q_S is $\frac{1}{2}$ times the winding number of the unit vector field $\hat{\mathbf{n}}$, see [12].

The gauge transformation U_0 can be decomposed as $U_0 = U_2 U_1$ where $U_2 = e^{i\varphi\sigma^x} e^{i\vartheta\sigma^y/2} e^{-i\varphi\sigma^x}$ and $U_1 = e^{i\varphi\sigma^x}$. In particular, transformation with U_1 brings the monopole part of \mathbf{A} to the form which has the Dirac string at negative z axis and is thus analogous to the Dirac description of the Abelian monopole [1]. The meaning of the different gauges can be understood qualitatively by considering the following ansatz $\psi_\pm(\mathbf{r}) = \phi(\mathbf{r})\zeta_\pm$ with $\zeta_\pm = (1/\sqrt{2}, \mp 1/\sqrt{2})^T$ for the condensate order parameter in the non-singular gauge $\mathcal{G}_2 = (\mathbf{A}', \Phi')$. For this simple trial wavefunction one obtains the following diagram

$$\begin{array}{ccc} \mathcal{G}_2 & & \mathcal{G}_1 \\ Q_M^{(2)} = 0 & \xrightarrow{U_2^\dagger} & Q_M^{(1)} = \pm 1 \\ Q_S^{(2)} = 0 & & Q_S^{(1)} = \pm 1 \end{array} \quad \xrightarrow{U_1^\dagger} \quad \begin{array}{c} \mathcal{G}_0 \\ Q_M^{(0)} = 0 \\ Q_S^{(0)} = 0, \end{array} \quad (13)$$

where $\mathcal{G}_0 = (\mathbf{A}, \Phi)$ is the original gauge given by Eqs. (2) and (3) and \mathcal{G}_1 is obtained from \mathcal{G}_0 with U_1 . In the next section we will use the intermediate gauge \mathcal{G}_1 to classify different stationary states.

Numerical analysis. — To study the lowest energy states we search numerically solutions to the field equations corresponding to the Hamiltonian in Eq. (4). We discretize the Hamiltonian using the scheme previously utilized in the lattice gauge theory studies of the non-Abelian Higgs model [18]. We replace $(D_\mu\psi)^\dagger(D_\mu\psi)$ by $C_{\mu,X} = (\psi_{X+\hat{\mu}} - U_{\mu,X}\psi_X)^\dagger(\psi_{X+\hat{\mu}} - U_{\mu,X}\psi_X)/a^2$, where $X = (u, v, w) \in \mathbb{N}^3$ denotes one lattice site, $U_{\mu,X} = e^{iaA'_{\mu,X}}$ is the link variable, and $\hat{\mu}$ is one lattice unit in the direction μ in the lattice. When the lattice

constant a tends to zero, $C_{\mu,X}$ reduces to $(D_\mu\psi)^\dagger(D_\mu\psi)$. The rest of the Hamiltonian is readily discretized yielding the mean field energy

$$\mathcal{E}_d = \frac{\hbar^2}{2m} \sum_{X,\mu} C_{\mu,X} + \sum_X [\psi_X^\dagger (V_{\text{ext},X} + \Phi_X) \psi_X + \frac{c_0}{2} |\psi_X|^4]$$

Minimizing the free energy $\mathcal{F}_d = \mathcal{E}_d - \mu_{\text{ch}} \sum_X |\psi_X|^2$ gives the discrete version of the time-independent Gross–Pitaevskii (GP) equation

$$\begin{aligned} & -\frac{\hbar^2}{2ma^2} \sum_{\mu=x,y,z} (U_{\mu,X}^\dagger \psi_{X+\hat{\mu}} - 2\psi_X + U_{\mu,X-\hat{\mu}} \psi_{X-\hat{\mu}}) \\ & + (V_{\text{ext},X} + \Phi_X) \psi_X + c_0 |\psi_X|^2 \psi_X = \mu_{\text{ch}} \psi_X. \end{aligned} \quad (14)$$

The chemical potential guaranteeing the conservation of the particle number is denoted by μ_{ch} .

For the numerical calculation we assume the external potential of the form $V_{\text{ext}} = m\omega_r^2 r^2/2$ and for simplicity that the wave vectors of the laser fields are equal and $k = k' > 0$. The spatial variables are scaled with the harmonic oscillator length $a_{\text{ho}} = \sqrt{\hbar/m\omega_r}$. The strength of the inter-particle interaction is taken to be $c_0 N/\hbar\omega_r = 100$. We solve the GP equation (14) iteratively using the successive over-relaxation scheme (SOR) with periodic boundary conditions in a grid of 121^3 points. Since the vector potential in Eq. (5) is not well defined at the z axis we shift the grid in x -direction by a positive constant η taken to be $\eta = a/10$. The results are independent of the choice of η . The mean field energy of different stationary states is shown in Fig. 1 as a function of ka_{ho} . The calculations are carried out in the non-singular gauge $\mathcal{G}_2 = (\mathbf{A}', \Phi')$ for which the corresponding charges are denoted by $Q_S^{(2)}$ and $Q_M^{(2)}$.

All stationary states have $Q_S^{(2)} = Q_M^{(2)} = 0$ for $ka_{\text{ho}} \lesssim 3.5$. For $ka_{\text{ho}} \gtrsim 3.5$ the ground state contains a vortex and the classification in terms of Eq. (12) is no longer valid. For $ka_{\text{ho}} \lesssim 0.055$ there is a pair of solutions which are degenerate within the numerical accuracy. These solutions behave under the gauge transformation U_2^\dagger such that $Q_S^{(2)} = 0 \rightarrow Q_S^{(1)} = \pm 1$, where the charges in the intermediate gauge \mathcal{G}_1 are denoted by $Q_S^{(1)}$ and $Q_M^{(1)}$. For $ka_{\text{ho}} \gtrsim 2.1$ the pair appears again as excited states with $Q_S^{(1)} = 1$ state having slightly larger energy. The difference in the energies between these two states is, however, very small and cannot be easily distinguished from Fig. 1. In the original gauge \mathcal{G}_0 , we find $Q_S^{(0)} = 0$ for all found states, similar to Eq. (13). For $ka_{\text{ho}} \lesssim 3.5$ the lowest energy state is always the state with $Q_S^{(1)} = 0$.

In the gauge \mathcal{G}_0 , the single-particle operator \hat{h}_0 of the mean field Hamiltonian $\mathcal{H} = \int d\mathbf{r} [\psi^\dagger \hat{h}_0 \psi + \frac{c_0}{2} |\psi|^4]$ has the symmetry $\hat{h}_0(\varphi) = \sigma^z \hat{h}_0(-\varphi) \sigma^z$. The two-fold symmetry explains the degeneracy of $Q_S^{(1)} = \pm 1$ states for small ka_{ho} and suggests that the small energy difference

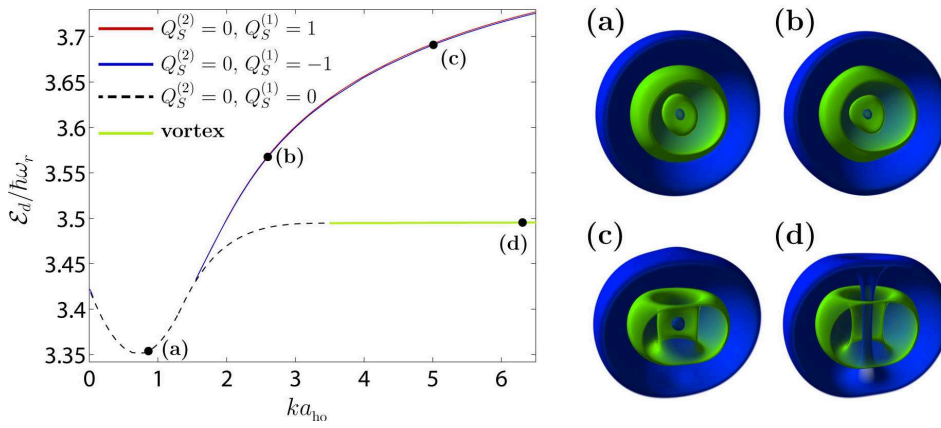


FIG. 1: (Color online) Left panel: mean field energies corresponding to the solutions of the Gross-Pitaevskii equation. States with $Q_S^{(1)} = \pm 1$ are practically indistinguishable in the plot. Right panel: isosurfaces of the particle densities for $ka_{ho} = 0.86$ (a), $ka_{ho} = 2.6$ (b), $ka_{ho} = 5.0$ (c), and $ka_{ho} = 6.3$ (d). The corresponding points are marked with \bullet in the energy diagram. In panel (d) the ground state has a vortex. The particle density increases from dark (blue) to light (green) color.

between these two states for large ka_{ho} can be due to the discrete lattice that breaks the symmetry with respect to rotations about the z axis. For small ka_{ho} , the energies of $Q_S^{(1)} = \pm 1$ and $Q_S^{(1)} = 0$ states almost coincide and they are indistinguishable in Fig. 1. The transition from the ground state with $Q_S^{(1)} = 0$ to the vortex ground state occurs when initially far apart separated pair of coreless vortices starts to become more and more tightly bound with increasing ka_{ho} eventually forming a singular vortex. Once the vortex state is formed, its energy seems to depend only weakly on ka_{ho} .

Within the mean field theory employed here, we cannot say anything definite about the possible fragmentation of the condensate. If the possible fragmentation is in this case related to restoring the broken symmetry [17], the true ground state could correspond to a fragmented condensate due to the two-fold symmetry of the mean field Hamiltonian. Numerically, however, we seem to find only a non-degenerate ground state. To study whether the BEC in a monopole field is fragmented, one needs to perform a calculation analogous to the one in Ref. [16].

Assuming that the underlying system consists of ^{87}Rb atoms, the two obvious gauge invariant observables are the particle density ρ and the hyperfine spin of the constituent atoms. Both of these quantities can be imaged accurately with the state-of-the-art techniques. In the original gauge \mathcal{G}_0 , the pseudospin \mathbf{s} can be expressed in term these two quantities which in principle enables reconstruction of \mathbf{s} . Thus it should be experimentally possible to distinguish between states with $Q_S^{(1)} = \pm 1$ and $Q_S^{(1)} = 0$ by performing a local change of basis using U_1 . In the current experiments with ^{87}Rb , one typically has $1.5 \lesssim ka_{ho} \lesssim 10$ for which the ground state either has $Q_S^{(1)} = 0$ or contains a vortex. For very tight traps with ω_r of order of kilohertz, the point where $Q_S^{(1)} = \pm 1$ and $Q_S^{(1)} = 0$ states start to coexist can be reached. By changing the trap parameters it is then possible to tune the value of ka_{ho} and explore different regions of the mean field phase diagram in Fig. 1. In particular, changing ω_r non-adiabatically could result in the $Q_S^{(1)} = \pm 1$ state.

In conclusion, we have studied a non-Abelian monopole in a Bose-Einstein condensate and showed that the system can be described with an effective $U(2)$ gauge invariant model. We identified a gauge invariant charge characterizing the system and classified different stationary states using this charge. Numerical calculations showed that the existence of a monopole in the non-Abelian gauge potential gives rise to a pseudospin texture with a topological charge that cancels the monopole charge.

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[19] The Greek indices $\mu, \nu \dots$ refer to the spatial coordinates and α runs over values $0, \dots, 3$. Indices $a, b \dots$ refer to the Pauli matrices $\boldsymbol{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$. Summation over re-

peated indices is implied.